Engaging Voluntary Contributions in Online Communities: A Hidden Markov Model

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Abstract

User contribution is critical to online communities but also difficult to sustain given its public goods nature. This paper studies the design of IT artifacts to motivate voluntary contributions in online communities. We propose a dynamic approach, which allows the effect of motivating mechanisms to change across users over time. We characterize the dynamics of user contributions using a hidden Markov model (HMM) with latent motivation states under the public goods framework. We focus on three motivating mechanisms on transitioning users between the latent states: reciprocity, peer recognition, and self-image. Based on Bayesian estimation of the model with user-level panel data, we identify three motivation states (low, medium, and high), and show that the motivating mechanisms, implemented through various IT-artifacts, could work differently across states. Specifically, reciprocity is only effective to transition users from low to medium motivation state, whereas peer recognition can boost all users to higher states. And self-image shows no effect when a user is already in high motivation state, although it helps users in low and medium states move to the high state. Design simulations on our structural model provide additional insights into the consequences of changing specific IT artifacts. These findings offer implications for platform designers on how to motivate user contributions and build sustainable online communities.

Keywords: Online community, IT artifacts, voluntary contribution, dynamics of contribution, motivating mechanisms, structural modelling, public goods, hidden Markov model, Bayesian estimation

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Introduction

In the growing digital economy, online communities have become central in bringing together large numbers of geographically dispersed individuals to spark novel ideas, collaborate on inventions, and share knowledge (Boudreau and Lakhani 2009; Goh et al. 2016; von Hippel 2005). Users in many of these communities, e.g., knowledge sharing platforms like StackExchange and Quora, usually contribute voluntarily without receiving monetary compensation. To engage user contributions in such a setting, online communities commonly design information technology (IT) artifacts as motivating mechanisms (Ma and Agarwal 2007; Ray et al. 2014). Among these motivating mechanisms, various forms of "gamification", such as badges, votes, and status systems, have been widely adopted (Zichermann and Cunningham 2011).

Despite their growing popularity, such design efforts do not always achieve positive results. Most of the online communities (even famous ones like Wikipedia) face the challenge of *declining* user participation over time (Simonite 2013). In particular, inappropriate design could alienate users and drive them away. For instance, Wikipedia's quality control mechanism was counterproductive in retaining new users (Halfaker et al. 2013). In the case of the social news aggregator Digg.com, the redesign efforts angered users and lost them to competitors (Metz 2012). While social comparison can encourage users below the median to contribute more, users above the median could decrease their contribution to conform to the social norm (Chen et al. 2010). Goal-setting can induce users to exert efforts, but it can also reduce efforts after users reach the goals (Goes et al. 2016).

From a theoretical perspective, the design of such mechanisms is related to individual motivations, which are extensively studied in the literature (e.g., Ma and Agarwal 2007; Porter and Donthu 2008). Yet, these motivations are usually theorized as *static*, even though recent research suggests that users' contribution patterns exhibit significant *dynamics* (Sauermann and Franzoni 2015). Fitting a static model to data generated by a dynamic process may result in misleading findings. To the best of our knowledge, no prior work has empirically analyzed the *dynamic* relationship between motivating mechanisms and voluntary user contribution in online communities. This leaves a gap in our understanding of how a user's motivation and contribution are *dynamically* influenced by the design of motivating mechanisms.

In this paper, we focus on the design of motivating mechanisms through IT artifacts when user motivations can change, and investigate the following research questions: (1) what kinds of mechanisms and IT artifacts are effective to transfer users among different levels of motivation? (2) how much would users contribute given their levels of motivation? An understanding of the *dynamic* effect along these two dimensions is essential to better design IT artifacts and effectively motivate user contributions.

In contrast to the literature, we take a *dynamic* approach. Specifically, we propose a structural econometric model, in which we integrate a *hidden Markov model* (HMM) into the public goods framework. This structural approach characterizes the dynamics of user contributions with different motivation states, as well as and the *transition* between the states. With a unique panel data set collected from a knowledge-sharing community, we use Bayesian estimation to jointly estimate the effect of motivating mechanisms on transition probabilities between the states, and user contributions conditional on their motivation states.

We find that the same motivating mechanism could work differently across states. For example, reciprocity is only effective to transition users from low to medium motivation state, whereas peer recognition (such as votes and acceptance from other users) are effective to *elevate* all users to the high motivation state. Badges are effective to transfer a low- or medium-motivation user to the high motivation state, but surprisingly, they show no effect when a user is already in the high motivation state. We also find that users do respond more to the demand of knowledge (i.e., number of questions that match their expertise) when they are in higher motivation states. These results provide important implications for the design of IT artifacts in online communities, and open up a new area for community managers to explore.

Our research has several features. First, we advance the literature from the conventional static approach to a *dynamic* perspective on voluntary user contributions, bringing about managerial insights unavailable in prior studies. Second, our structural model helps advance the modelling approach in the online community literature, as it explicitly characterizes the dynamics of user contributions at the individual level. Third, our dynamic approach provides more nuanced insights into an increasingly important mode of open collaboration, and is applicable to a wide range of online communities where user contributions are voluntary and fluctuate over time (Xu et al 2012).

Literature Review

We draw on the literature to build a theory of dynamic motivation and contribution in online communities. We first examine factors that affect user motivation, and illustrate how the design of IT artifacts can influence contribution through various motivations. Then we identify the dynamics of motivation and contribution as a gap in the literature, which motivates our hidden Markov model to characterize such dynamics.

User Motivation and IT Artifacts as Motivating Mechanisms

In online communities, motivation is the key driver of user contribution. The literature has distinguished three types of motivations: intrinsic, extrinsic, and internalized extrinsic motivation (see von Krogh et al. 2012 for a review). Intrinsic motivation stems from intrinsic benefits, such as joy, fulfilment, and self-efficacy (Kankanhalli et al. 2005; Ray et al. 2014). In contrast, extrinsic motivation is driven by economic rewards such as career prospects (Huang and Zhang 2016; Roberts et al. 2006).

Internalized extrinsic motivation is unique, in that it arises from external influences at first, but users can assimilate these influences and perceive them as self-regulating behavior rather than external impositions (Deci and Ryan 2002). In online communities, internalized extrinsic motivation includes reciprocity and reputation (von Krogh et al. 2012). Reputation can be further classified as peer recognition and self-image. We discuss these three factors below.

First, reciprocity suggests that users who have received others' help tend to return the favor. It is shown to drive contributions in open source software (Lakhani and von Hippel 2003; Zhu and Zhou 2012) and online communities (Chiu et al. 2006). Second, social interactions, especially peer recognition, validate users that their role in the community is expected (Ray et al. 2014). This identity-verification process enhances the confidence of contributors, and reassures users to contribute with their unique identity (Ma and Agarwal 2007). Third, concerns over self-image can also motivate contribution, as people care about the way others perceive them (Bénabou and Tirole 2006). In online communities, it is common that users contribute in order to earn respect from others and build a better image (Kankanhalli et al. 2005). It is also shown that self-image is important to drive participation in social media (Toubia and Stephen 2013).

Through its influence on internalized extrinsic motivation, an online community can affect users' contribution by employing various IT artifacts as motivating mechanisms (Ma and Agarwal 2007; Peng and Dey 2013). Examples of such IT artifacts include points, badges, status, reputation systems, and other features that facilitate verification of self-identity (e.g., Khansa et al. 2015). Among various IT artifacts, we focus on three types of mechanisms that support reciprocity, peer recognition and self-image, specific to our research context of knowledge-sharing online communities. First, users whose questions have been answered by others may be more likely to answer others' questions in return. Second, users may care the evaluation from their peers, through IT artifacts that facilitate user interactions, such as up-votes and acceptance of answers. Third, the community awards badges to users when they contribute, which improves users' self-image and serve as a signaling mechanism.

Dynamics of Motivation and Contribution

Despite the growing literature on user motivations and motivating mechanisms, a majority of these studies build on an implicit *static* assumption. That is, the relationship between motivating mechanisms and user contributions does not change over time. This is a strong assumption, especially if we examine user contributions over a long time. It is because user motivations often evolve with their changing personal characteristics and their interactions with the community via the channel of various IT artifacts, leading to the fluctuation of contributions (Franzoni and Sauermann 2014).

The internalized extrinsic motivation and the facilitating IT artifacts may have different effects when user motivations are evolving. We expect reciprocity to enhance user motivation. But this effect may be less salient for highly motivated individuals because they contribute disproportionally more than they receive. For peer recognition, users may feel satisfied with their current good reputation, and do not contribute further. Indeed, research finds that users may decrease their contributions after reaching certain incentive hierarchy (Goes et al. 2016). With large amount of badges, users may suffer from the "moral licensing" effect, where people may feel justified behaving non-prosocially when they have done something pro-social (Gneezy et al. 2012). Having contributed to the online community and been endorsed by badges, users may sit on their laurels and feel entitled not to contribute subsequently.

Modeling the dynamic process in a static way leads to problematic estimation and misleading implications. In this paper, we propose a theoretical framework of dynamics that differs from the literature in two dimensions. First, we propose *motivation state* as a general construct to characterize individual's propensity to contribute, and model it as a mediator between the motivating mechanisms and user contributions. Second, we relax the assumption that an individual's motivation state is fixed, and allow it to change over time. Our model also allows the impact of motivating mechanisms to be heterogeneous when users are in different motivation states. As such, we introduce a general model to explain the dynamics of user contributions.

Further, our approach has an evident empirical advantage. Prior studies use survey data to measure the psychological state of contributing (e.g., Ray et al. 2014). However, these constructs are hard to quantify with consensus. It is also costly to survey a large number of users over time to reveal the dynamics. Instead, we use observational data to infer motivation states and characterize individual dynamics. This approach enables community managers to estimate the dynamic motivation states of all users.

We make a distinction between the dynamics at the community level and at the individual user level. At the community level, the dynamics may come from membership turnover (Butler 2001). Recent studies find that more turnover may be better for the community at the knowledge-retention stage of the life cycle (Ransbotham and Kane 2011). However, it remains unclear how dynamics at the aggregated level may come from individual-level behaviors, and what mechanisms community designers can use to promote the desired outcome. To narrow this gap, we focus on the dynamics of user motivation and contribution at the individual level.

Model the Dynamics of User Contribution

One challenge of capturing individual-level dynamics is that the structure of such dynamics is usually unobservable. To capture this latent structure, the discrete state space model is a useful approach in the literature (e.g., Heckman 1981). For example, an individual's present decision depends on his past decision. In most of these models, the states are observable (e.g., brand switching of customers). Still, they tend to ignore other dynamics that could contribute to the change of states. In many other scenarios, however, we cannot observe the underlying states that drive the individual-level dynamics, e.g., motivation states in our research context. In this case, the hidden Markov model (HMM) can be useful.

An HMM is a stochastic process that consists of three elements: a finite set of hidden states, observed outcomes conditional on the hidden state, and the probabilities of transitioning from one state to another. It has wide applications in modelling stock market volatility (e.g. Rydén et al. 1998), business cycles (e.g., Hamilton 1989), customer relationship management (Netzer et al. 2008), and the mental states of patients in healthcare communities (Yan and Tan 2014). As far as we are aware, HMM has not yet been applied to modelling user contributions in online communities. Furthermore, we incorporate HMM into the public goods model (Bénabou and Tirole 2006) to formalize the dynamic effect of motivating mechanisms. This structural modeling approach allows us to explicitly characterize the dynamics of user contributions at the individual level.

Research Design

To characterize the dynamics of user contributions, we develop a HMM model as shown in Figure 1. It illustrates how a user could switch between motivation states through various motivating schemes, and how his contribution probability depends on the states. Specifically, our HMM model has three elements:

(1) We model users with different hidden *motivation states*, with 1 being the lowest and J the highest. The state captures the strength of motivation to contribute. At any time t, a user is in only one state.

(2) From time *t-1* to *t*, the user could switch to any state with certain probability, which is affected by the user's interaction with the community, such as how his contribution is evaluated by the peers. The community interactions are enabled by various IT artifacts that work through motivating mechanisms (e.g., reciprocity, peer recognition, and self-image).

(3) Conditional on his state in *t*, a user may respond differently to community and individual characteristics (e.g., size of the community and the demand for knowledge). We can observe this state-dependent response as his level of contributions in *t*.

This model can be applicable to various online communities, as long as user motivation is unobservable, user contributions fluctuate, and the goal of the community managers is to motivate user contributions by designing appropriate IT artifacts.

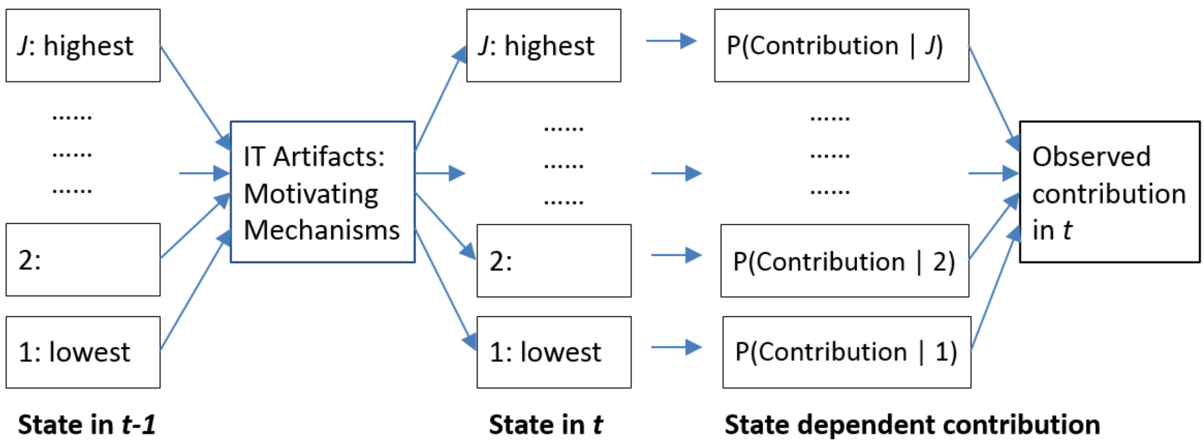

Figure 1. Hidden Markov Model of User Contributions

Research Context

We study our research questions in an online community called StackExchange (stackexchange.com), which is a representative, large network of knowledge-sharing platform based on Wikipedia-style voluntary contributions. It started in 2008 with StackOverflow, a knowledge sharing website on programming. Now it has expanded to more than 100 sub-sites covering widespread technical (e.g. math, Tex) and non-technical (e.g. cooking, bicycle) topics. On each sub-site, users ask topic-related questions and provide answers. Users can also vote, comment, and revise other users' questions and answers as they do in Wikipedia, which allows the community to improve the content collectively.

Like many other online communities, StackExchange faces the challenge to maintain user participation. To cope with this, StackExchange employs various mechanisms to encourage user contribution and to sustain the high quality of questions and answers. For example, when a user receives 10 up-votes on one of his answers, he earns a "Nice Answer" badge. If the answer receives more than 40 up-votes and is accepted by the question poster, he is rewarded a "Guru" badge. Our sample includes 158 types of badges and they have been awarded for 414,761 times. A user can also earn reputation points, which are displayed together with the badges right below the user name on the profile page. These mechanisms serve as important channels for the users' identity verification.

These features help us understand user behaviors when knowledge collaboration is organized in such a voluntary community. First, it provides detailed data about user interactions. For example, we can observe when a user receives an up-vote on his answer, and whether his answer has been accepted. Such fine-grained user-level data help us identify the effect of different interactions on users' transition probabilities. Second, as many other online communities are using similar motivating mechanisms, our analysis could be generalized in a broader sense. For example, peer voting is used in crowdsourcing ideation initiatives (Huang et al. 2014), and the badge system is one important device in many online communities (Piskorski et al. 2010).

Data

Our data comes from SuperUser.com, a sub-site of StackExchange, for computer enthusiasts and power users. We employ SuperUser because of its data quality, as it is one of the largest sub-sites on StackExchange by the number of contributions. It employs various mechanisms to engage users, such as voting and badge systems. Hence the site has rich information on user interactions that are appropriate to study our research questions.

SuperUser was launched in July 2009, and has accumulated about 214,000 questions and over 351,000 answers by April 2014. We collected detailed data on daily activities of each user from July 12th 2009 to March 1st 2012 (964 days). We only include users who contributed at least 10 answers during the sample period.[1] Our full sample contains 2,147 users who have contributed 127,360 out of the 157,375 answers, equivalent to 26,200 hours of work.[2] Because these users make over 80% of the contributions, it is critical to understand their behaviors.

Community and User Level Trends

We first demonstrate the general trends of the data in Figure 2. Except the surge around the launch of the community, the numbers of new questions (graph-a) and answers (graph-b) are relatively stable over time. Similarly, the trends are stable for the numbers of badges and up-votes, as shown in graphs (d) and (e), respectively. Graph (c) plots the number of accepted answers each day. The stable trend suggests that question posters deem the quality of answers being consistent over time. We also plot in graph (f) the average up-votes per answer, which indicates the overall quality of the answers in the community. The trend is stable except for a decline at the initial stage. Overall, SuperUser is a relatively healthy community with steady contributions in our sample.

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[1] We also conduct robustness checks with different samples. See the Robustness Checks Section.

[2] Assuming each answer takes 10 minutes on average. This estimate is conservative since many users would need to do some coding in order to provide an answer, which may take more time.

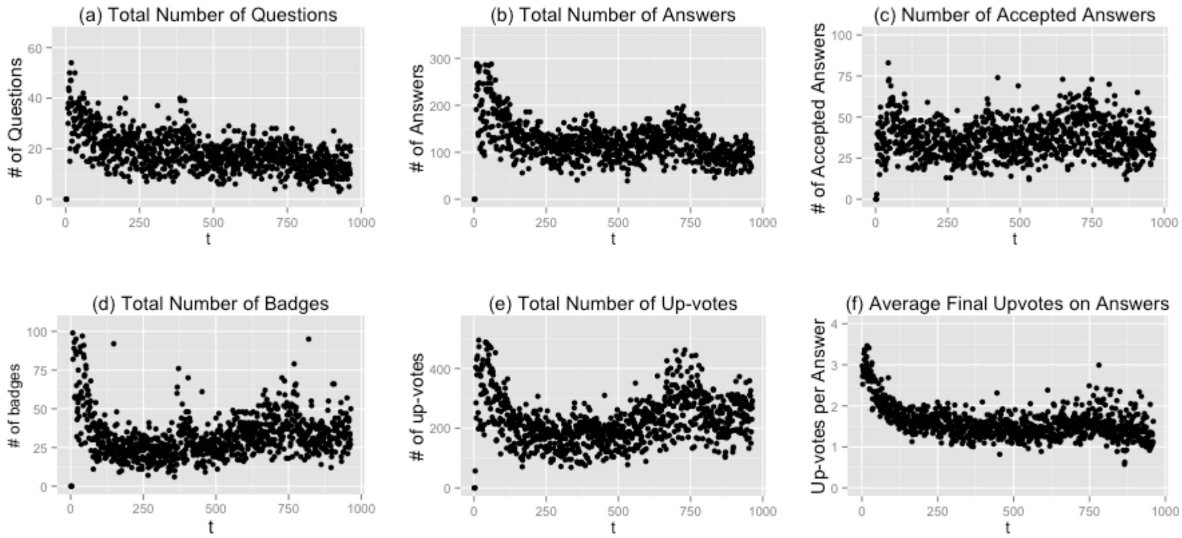

Figure 2. Trends of Key Variables

The contributions at the individual level, however, show a different pattern. Figure 3(a) presents the average number of answers contributed by each user over time. The contribution shows a *declining* pattern. However, some users stay for a long time in the community. Figure 3(b) shows a histogram of contribution tenure, which is defined as the days between the first and last answers of each user. We can see significant heterogeneity in the time span during which users contribute. For those users contributing for a long time, understanding their behaviors and motivations can help develop sustainable online communities.

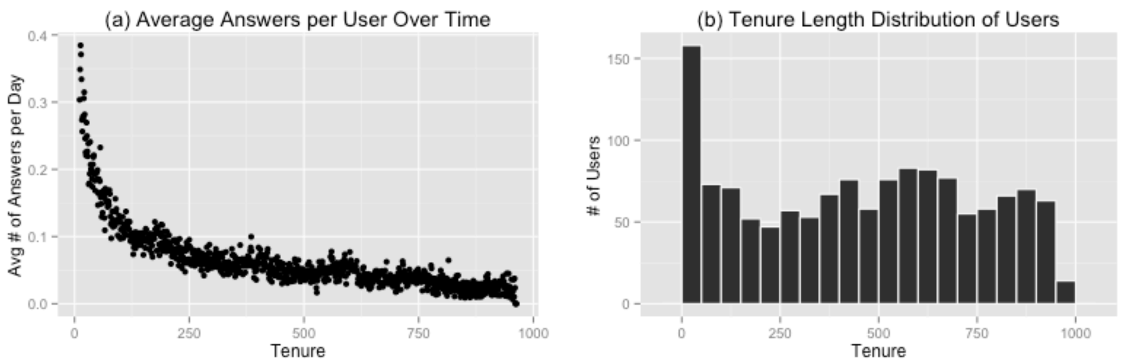

Figure 3. Average Answers over Time and Tenure Distribution

We further drill down to individual level and plot the contributions of five representative users from our sample in Figure 4 (user IDs anonymized). Each row shows the answers of a user over the sample period. Each point represents the number of answers contributed by that user. A point is missing if the user does not contribute at a particular time. We observe that even relatively active users exhibit substantial fluctuation of contributions during their tenure. They actively contribute for some time periods, while idling for other periods. Our goal is to model the fluctuation of user contributions (dynamics), and study the influence of different motivating mechanisms that drive such dynamics of active (or lack of) contributions.

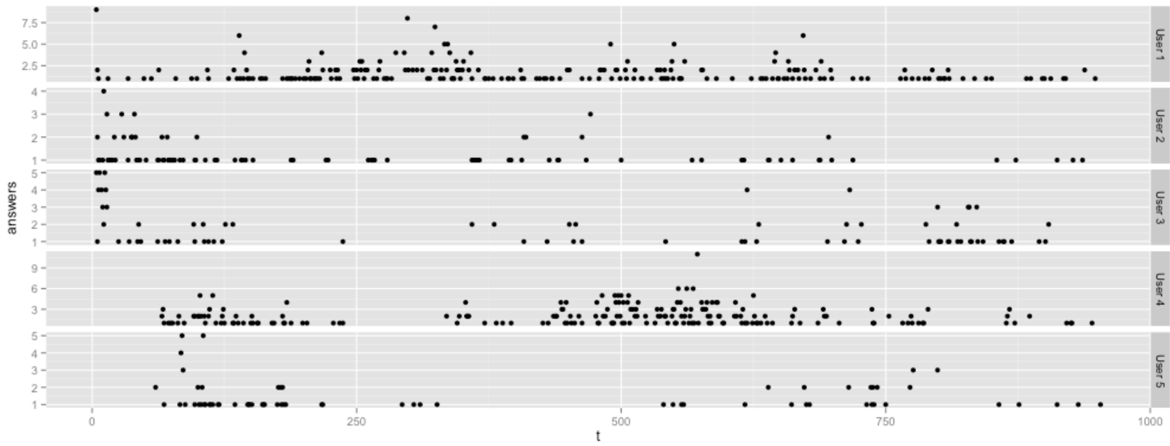

Figure 4. Fluctuation of User Contribution over Time

Model Development: Structural Modelling of User Behavior

In this section, we describe the details of our structural model with HMM, where a user interacts with the community and decides his level of contribution.

Modeling User Contribution as Public Goods

In online communities, user contributions are public goods in nature, because they are voluntary, free and open. The key issue about public goods is free-riding, which means that everyone can share the benefits, but only the contributors incur the cost. Then under-provision is a common equilibrium in many pure altruism models (e.g., Andreoni 1988). It follows that online communities may eventually be depleted, suffered from the "tragedy of the commons." But these models are not adequate to explain why large groups, such as the Wikipedia community, are able to attract substantial user contributions. Such discrepancy between theoretical models and empirical phenomena may be reconciled by *impure altruism models* (e.g., Andreoni 1990; Benebou and Tirole 2006), where individuals contribute because they obtain utilities not only from pure altruism, but also from their own private benefits, such as signalling personal skills or the fulfilment of helping others.

We use the public goods framework, particularly the *impure altruism models*, to model user contributions in an online community. Each self-interested user chooses how much to contribute. A user's net utility consists of three parts: (1) his valuation of the accumulated contribution (e.g., knowledge) in the community, (2) his valuation of his own contribution, and (3) his cost of contribution. The first part captures the benefit the user could obtain from the community, as suggested by the pure altruism literature. The second part captures the impure altruism, corresponding to internalized extrinsic motivations that we have reviewed in the Literature Review Section. The third part suggests that making contribution is costly in terms of time and effort.

Assuming additive separability of the above three parts, we specify the utility function of user i at time t as:

$$U_{it}(Y_{it}, X_{it}, W_{i,t-1}, Y_{j\tau}) = \gamma_i \sum_{\tau=1}^{t} \sum_{j=1}^{N_\tau} \delta^{t-\tau} Y_{j\tau} + f(X_{it}, W_{i,t-1}) \cdot Y_{it} - \frac{1}{2} c_i Y_{it}^2 , \qquad (1)$$

where Y_{it} is the contribution of user i at time t. Intuitively, a user gains utility from the accumulative knowledge in the community, and his own incremental contribution at present, net of his cost.

We choose such a functional form following Chen et al. (2010). The first term on the right hand side captures user i's valuation of the accumulated contribution of the community, in which γ_i is user i's marginal benefit from the accumulated contribution, N_τ is the number of users in the community at time τ, and δ is a discount factor of the contribution stock. In the second term, $f\left(X_{it}, W_{i,t-1}\right)$ captures user i 's valuation of his own contribution at time t. This could be viewed as a parsimonious version of the "image rewards" as in the prosocial behavior model (Bénabou and Tirole 2006), which will "depend on the informational and economic context, including what others are doing." Therefore, this valuation could change over time with X_{it}, which is a vector of individual and community characteristics, and $W_{i,t-1}$, which captures the community interactions in a vector. Essentially, in our model a user's valuation of his own contribution could fluctuate because of his changing interactions with peers in the community. The third term is the cost function. We use a quadratic cost function to capture the convex cost of contributions (Gu et al. 2007).

Using the first order condition of Equation (1) with respect to Y_{it}, we obtain the equilibrium contribution of user i at time t as:

$$Y_{it}^{*} = \frac{\gamma_i + f\left(X_{it}, W_{i,t-1}\right)}{c_i}. \tag{2}$$

For analytical tractability, we assume that $f(X_{it}, W_{i,t-1})$ is linear in X_{it} conditional on motivation state s_{it}. And s_{it} is further determined by a latent transition propensity $L(W_{i,t-1})$ (details in the Transition Probabilities of Motivation States Section, below). We then obtain:

$$Y_{it}^{*} = X_{it}'\beta_{s_{it}} + \varepsilon_{it}, \qquad (\varepsilon_{it}|X_{it}, s_{it}) \sim N(0, \sigma^2) \tag{3}$$

where X_{it} is a vector of community and individual characteristics, the error term ε_{it} follows a normal distribution with mean zero and variance σ^2, and the individual marginal benefit parameter γ_i and cost coefficient c_i are represented by a constant term and time-varying individual characteristics in X_{it}, and the error term ε_{it}.

Our goal is to estimate the coefficient vector $\beta_{s_{it}}$, which captures the influence of vector X_{it} on the user's contribution Y_{it}. Note that vector $\beta_{s_{it}}$ depends on user i's motivation state s_{it}, which is associated with user i's previous interactions with the community $W_{i,t-1}$. We detail the motivation states and their transitions in our model below.

Motivation States in HMM

Our proposed HMM characterizes the dynamics of a user's contribution as two stochastic processes: a process of observed contributions, and an underlying unobserved process of the user's motivation states. We denote s_{it} the state of user *i* at time *t*. A user can have *J* hidden motivation states: $s_{it} \in S = \{1, 2, \ldots, J\}$.

The hidden state captures the time-dependent feature of a user's valuation of his own contribution, i.e., the strength of his motivation to contribute. If a user has high valuation of the contributions he provided to the community at time *t-1* (i.e., in a high motivation state), he may also highly value his contributions at time *t*. Based on his state, a user responds differently to the community and individual characteristics (i.e., vector X_{it}). For example, if a user is in a high motivation state, he may be more likely to respond to new questions posted in the community. The observed contributions could be regarded as a noisy signal of the hidden state process. The hidden state and observed contributions together form a hidden Markov chain (Rabiner 1989).

From time *t-1* to *t*, a user may stay in one state, or switch to another. In our HMM, the state process $\{s_{it}\}_{t \geq 0}$ is characterized as a first-order Markov chain with state space $S = \{1, 2, \ldots, J\}$. Together with Y_{it}, the observed contribution of user *i* at time *t*, we can model the vector-valued stochastic process (Y_{it}, s_{it}) as a hidden Markov chain. Its probability of transition from one period to the next can be factorized as:

$$P\left(Y_{it}, s_{it} \middle| Y_{i,t-1}, s_{i,t-1}\right) = P(Y_{it}|s_{it}) \cdot p\left(s_{i,t-1}, s_{it}\right),$$

where $p(s_{i,t-1}, s_{it})$ is the transition probability from state $s_{i,t-1}$ to state s_{it}, and $P(Y_{it}|s_{it})$ is the conditional probability describing the state-dependent contributions. We elaborate these two probabilities in the next two sub-sections, respectively.

Transition Probabilities of Motivation States

A user can switch among all the possible states in S. The transition matrix $P(s_{i,t-1}, s_{it})$ below characterizes the probability of such transitions:

$$P(s_{i,t-1}, s_{it}) = \begin{bmatrix} p(1,1) & p(1,2) & \dots & p(1,J) \\ p(2,1) & p(2,2) & \dots & p(2,J) \\ \vdots & \vdots & \ddots & \vdots \\ p(J,1) & p(J,2) & \dots & p(J,J) \end{bmatrix}$$

where $p(j,k)$ is the transition probability from state *j* to state *k*, and $\sum_k p(j,k) = 1$ for all $j,k \in S$. We assume that $p(j,k)$ is influenced by a user's interactions with the community, which may create certain social or personal norms for the user (Bénabou and Tirole 2006). The user then evaluates his own contributions differently based on the norms. For instance, if all of his past contributions were voted up and appreciated, the user would be more likely to value his own contribution and remain highly motivated. Otherwise, he may switch to a lower motivation state.

We model the transition probabilities with a *probit* model (Wooldridge 2010). We assume that the *states* are determined by a latent propensity of transition L_{it}:

$$L_{it} = W'_{i,t-1}\xi_{s_{i,t-1}} + u_{it}, \quad \left(u_{it} \middle| W_{i,t-1}, s_{i,t-1}\right) \sim N(0, \sigma_u^2) \tag{4}$$

such that $s_{it} = j$ if $L_{it} \in [\mu_{j-1}, \mu_j)$, where $W_{i,t-1}$ is a vector of lagged variables related to the user's previous interactions with the community, $\xi_{s_{i,t-1}}$ is a vector of the corresponding coefficients, and u_{it} is a normal error term from the *probit* model. In this model, the $\{\mu_j\}, j = 1, \dots, J,$ are threshold values with μ_0 normalized to negative infinity, μ_1 to zero, and μ_J to infinity. The remaining cut-off points are assumed to satisfy $\mu_2 \leq \dots \leq \mu_{J-1}$ so that the cumulative probabilities are non-decreasing (Chib 2001). Note that $\xi_{s_{i,t-1}}$ is state-specific, capturing different effects of $W_{i,t-1}$ under different states. Then we obtain the transition probability as follows:

$$
\begin{aligned}
p(j,k) &= P\left(s_{it} = k \middle| s_{i,t-1} = j, W_{i,t-1}\right) \\
&= P\left(\mu_{k-1} \le L_{it} < \mu_k \middle| s_{i,t-1} = j, W_{i,t-1}\right) \\
&= P\left(L_{it} < \mu_k \middle| s_{i,t-1} = j, W_{i,t-1}\right) - P\left(L_{it} < \mu_{k-1} \middle| s_{i,t-1} = j, W_{i,t-1}\right) \\
&= \Phi\left(\frac{\mu_k - W'_{i,t-1}\xi_j}{\sigma_u}\right) - \Phi\left(\frac{\mu_{k-1} - W'_{i,t-1}\xi_j}{\sigma_u}\right),
\end{aligned}
\tag{5}
$$

where Φ is the standard normal distribution function. When a user first joins the community, we assume that he has an initial probability p_j to be in motivation state j and $\sum_{j=1}^{J} p_j = 1$.

State-Dependent Contributions

Given the states above, we now derive the conditional probability $(Y_{it}|s_{it})$ to describe the state-dependent contributions. Since the observed user contributions are non-negative, we adopt the standard *Tobit* model (Wooldridge 2010) following the Bayesian literature (Rossi and Allenby 2003):

$$
\begin{gathered}
Y^*_{it} = X'_{it}\beta_{s_{it}} + \varepsilon_{it}, \qquad (\varepsilon_{it}|X_{it}, s_{it}) \sim N(0, \sigma^2) \\
and \;\; Y_{it} = \max(0, Y^*_{it}),
\end{gathered}
$$

where Y_{it} stands for the observed contributions. Then the state-dependent contributions would follow the distribution below. The probability of making no contribution is

$$
P(Y_{it} = 0|X_{it}, s_{it}) = P(Y^*_{it} \le 0|X_{it}, s_{it}) = 1 - \Phi\left(\frac{X'_{it}\beta_{s_{it}}}{\sigma}\right).
$$

For $Y_{it} > 0$, the probability density function is

$$
f(Y_{it}|X_{it}, s_{it}) = \frac{1}{\sigma}\phi\left(\frac{Y_{it} - X'_{it}\beta_{s_{it}}}{\sigma}\right),
$$

where ϕ is the standard normal density function. With the transition probabilities and the state-dependent contributions specified, we now proceed to estimation and identification.

Analysis

Estimation and Identification

We estimate the state-dependent contribution parameters $\beta_{s_{it}}$ in equation (3), and the transition matrix coefficients $\xi_{s_{i,t-1}}$ in equation (4). Since $s_{it} \in S$, we essentially estimate the parameter vectors $\boldsymbol{\beta} = (\beta_1, \dots, \beta_J)$ and $\boldsymbol{\xi} = (\xi_1, \dots, \xi_J)$, where $\boldsymbol{\beta}$ captures the effect of community and individual characteristics on the contributions, and $\boldsymbol{\xi}$ captures the influence of community interactions on the user's state transition probability. To estimate these key parameters, we also estimate the standard deviations σ and σ_u, as well as the state process $\tilde{\boldsymbol{S}} = \{s_{it}\}, t = 1, \dots, T; i = 1, \dots, N_t$. For ease of reference, we write the parameter space as $\boldsymbol{\theta} = \{\boldsymbol{\beta}, \boldsymbol{\xi}, \sigma, \sigma_u\}$ and $\tilde{\boldsymbol{S}}$. Note that $\boldsymbol{\beta}$ and $\boldsymbol{\xi}$ are state-dependent, while σ and σ_u are not.

We estimate our HMM using a Bayesian procedure developed by Kim and Nelson (1999). The Bayesian estimation algorithm treats $\boldsymbol{\theta}$ and $\tilde{\boldsymbol{S}}$ as random variables with prior distributions. The algorithm then updates their joint distributions $\pi\left(\boldsymbol{\theta}, \tilde{\boldsymbol{S}} \middle| \boldsymbol{Y}, \boldsymbol{X}, \boldsymbol{W}\right)$ using Gibbs sampling (Albert and Chib 1993). This updates the posterior distribution by incorporating the observed information from data.[3]

Bayesian estimations of HMM models may encounter the "label switching" problem (Jasra et al. 2005), which means our posterior distribution of $\boldsymbol{\theta}$ and $\tilde{\boldsymbol{S}}$ may be invariant if we switch the labels. Since the *motivation states* in our context have self-evident economic interpretation, we adopt a normalization requirement that the constant terms in $\beta_j \in \boldsymbol{\beta}$ are ordered. Denote the constant term in β_j as c_j^x. We permutate $\boldsymbol{\beta}$ according to c_j^x such that $c_1^x \leq \cdots \leq c_J^x$ in each draw of our Gibbs samplers. This requirement means that without any stimulus, a user in a high motivation state on average contributes more than if he were in a

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[3] Technical details of the sampling algorithm are in Appendix A1.

lower motivation state. This technique helps us identify the states in our model.

Samples and Variables

To test our structural model, we construct a user-date panel of the 2,147 users in 964 days from SuperUser. We exclude the first 100 days (with substantial fluctuation), and analyze the steady-state periods afterwards.[4] Because of computational burden (over 2 million data points), we divide the sample into sub-samples that each contains 200 days. Our estimation focuses on the subsample in 101-300 days with 1,215 unique users, and 210,890 user-date observations. Table 1 presents the definitions of our variables and summary statistics. We use other sub-samples for robustness checks.

Table 1. Variables and Descriptive Statistics

Variable	Description	Mean	S.D.	Min	Max
Dependent Variable ($\boldsymbol{Y}_{it}$)					
$Answers_{it}$	Number of answers	0.118	0.657	0	23
Community and Individual Characteristics ($\boldsymbol{X}_{it}$)					
Matched_tags	Number of tags matched between questions and the user's profile	27.061	23.894	0	225
$Tenure_{it}$	Number of days since the user registered	153.537	73.142	0	299
$Total\_answers_{i,t-1}$	Total number of past answers by the user	31.405	88.596	0	2018
$Group\_size_t$	Number of participating users	112.400	21.659	54	158
Community Interactions ($\boldsymbol{W}_{i,t-1}$)					
$Answers\_received_{i,t-1}$	Number of answers to past questions received by the user	0.039	0.318	0	19
$Upvotes\_answer_{i,t-1}$	Number of up-votes to past answers of the user	0.177	1.006	0	37
$Accepted\_answers_{i,t-1}$	Number of accepted answers of the user	0.032	0.29	0	10
$Badges_{i,t-1}$	Number of badges earned by the user	0.022	0.177	0	10

Our dependent variable is $Answers_{it}$, which is the number of answers provided by user i at time t. We choose this dependent variable because among various ways to participate in the

[4] We would like to thank the AE and an anonymous reviewer for the suggestion of focusing on steady-state analysis.

community, providing answers may be the most crucial because of the knowledge-sharing nature of the site. It is also the most challenging activity as it takes time and effort and requires certain domain expertise.

We categorize two sets of explanatory variables that may affect users' transition probabilities (*W*) and conditional contributions (*X*), respectively. The variables in vector *W* contain individual and community characteristics enabled by IT-artifacts that could have an enduring effect on a user's motivation state. First, if a user's questions are answered by others, he may be more likely to return to the site and may have higher chance to contribute. Moreover, he may be more likely to answer others' questions out of reciprocity. We use the number of answers a user receives on his past questions ($Answers\_received_{i,t-1}$) to capture such reciprocity. Second, peer recognition can play a role in state transitions. When more answers provided by a user are voted up or accepted as the best answer, one may value his contribution higher because the contribution is appreciated by the community. This may transfer the user to a high motivation state so that he contributes even more. We measure these effects by the number of *up-votes* that a user receives on his previous answers ($Upvotes\_answer_{i,t-1}$), as well as the number of accepted answers of a user ($Accepted\_answers_{i,t-1}$). Third, self-image related motivation may also influence a user. To award users for their contributions, SuperUser grants users various badges, which serve as a signalling mechanism for a user's self-image. To examine the effect of the badge system, we include $Badges_{i,t-1}$ as another explanatory variable, which represents the incremental number of badges earned by user i for his answers at time $t-1$.

The variables in vector *X* contain individual and community characteristics that may have a direct effect on a user's valuation of his contribution. First, the types of questions are diverse in the community and user expertise is different; whether a user can contribute his knowledge depends on whether a question is within his domain.[5] StackExchange uses tags, i.e., certain words or phrases, to identify the topics of each question and the expertise of each user. By sorting questions and users into specific, well-defined categories, tags are a means of matching experts with questions that they are able to answer. We calculate $Matched\_tags_{it}$ as the number of identical tags matched between the questions and the user i at time t. $Matched\_tags_{it}$ captures not only the demand for knowledge on the site, but also the feasible supply of knowledge specific to the user's expertise.

Second, the tenure of membership can affect a user's contribution. As Figure 3(a) shows, users contribute less when they stay longer on the site. We use the days since user i registered ($Tenure_{it}$) to account for this declining tendency of contributions over time. Third, we also include the total number of answers that have been provided by the user ($Total\_answers_{i,t-1}$). The rationale is that if a user has provided more answers in the past, he may also be more inclined to provide new answers in the current period. Fourth, we proxy the community size by $Group\_size_t$, the number of users who participate in any activities at time t. Classic public goods models show that the average level of contribution *decreases* with group size, while in impure altruism models, the private benefits can increase with group size, as the enjoyment of contributing is enhanced by the number of recipients. We call this the *social effect*. As a group becomes larger, the motivation of pure altruism can decrease, while the social effect can increase. Given the importance of group size, we include it as a contextual factor in our analysis.

Model Selection

In our model specification, the number of states was not defined *a priori*. It instead needs to be estimated with the data. To estimate the number of states, we adopt several model

[5] We would like to thank the editors and reviewers for pointing out this important feature.

selection criteria from the literature. Our selection criteria include the log-likelihood, the commonly used Akaike information criterion (AIC) and Bayesian information criterion (BIC) (Singh et al. 2011, Yan and Tan 2014), and the Markov switching criterion (MSC) which is specially designed for Markov switching models (Netzer et al. 2008).[6] Given a set of candidate models for the data, the preferred model is the one with the minimum value of the selection criteria.

We estimate models of different states, and report the results in Table 2. Our benchmark is the one-state static model, which assumes that a user stays in the same motivation state throughout, and thus his contribution behavior does not change over time.[7] As Columns 2 – 5 show, whereas the static model has the largest value, the three-state HMM has the smallest value in each selection criterion. Hence, all selection criteria suggest that HMM models with more than one state are superior to the static model, and particularly the three-state HMM is the best-fitting model that outperforms other models. Therefore, we report estimation results of the three-state HMM hereafter.

Table 2. Selection of the Number of States

Number of states	- 2*Log-likelihood	AIC	BIC	MSC	Number of Variables
1	135,246.5	135,270.5	135,331.7	------	12
2	134,065.9	134,111.9	134,229.3	345,144.4	23
3	**119,391.6**	**119,461.6**	**119,640.1**	**330,957.1**	**35**
4	120,577.6	120,671.6	120,911.4	333,159.2	47

Estimation Results

Table 3 reports the estimated parameters of the three-state HMM based on Bayesian estimation. For ease of discussion, we refer the three motivation states as low, medium, and

6 The technical details of calculating the log-likelihood and selection criteria are in Appendix A2.

7 The static model follows the Heckman two-step estimation, with a participation equation and a contribution equation. We would like to thank an anonymous reviewer for suggesting this analysis.

high, denoted as L, M, and H, respectively. The coefficients in vectors $\boldsymbol{\beta}$ and $\boldsymbol{\xi}$ vary across states (the three columns), indicating that a change in states would lead to a change in contribution. The initial probabilities of being in L, M, and H states are 0.755, 0.216 and 0.029, respectively (bottom row). Hence, a new user tends to be in L state much more likely than in higher states. This confirms the importance of studying how to energize and motivate community members.

Table 3. Results of HMM Bayesian Estimation

Variable Name	State L (Low Motivation)	State M (Medium Motivation)	State H (High Motivation)
$\boldsymbol{X}_{it}$	$\boldsymbol{\beta}$ – Posterior Mean (Standard Deviation)		
c^x	-3.053*** (0.064)	0.316*** (0.086)	7.072*** (0.312)
$Matched\_tags_{it}$	0.015*** (0.000)	0.022*** (0.001)	0.029*** (0.001)
$Tenure_{it}$	-0.0004 (0.0003)	-0.006*** (0.003)	-0.016*** (0.001)
$Total\_answers_{i,t-1}$	0.001*** (0.000)	0.002*** (0.000)	0.003*** (0.000)
$Group\_size_t$	0.003*** (0.000)	0.002*** (0.000)	-0.010*** (0.002)
σ^2	1.011*** (0.005)		
$\boldsymbol{W}_{i,t-1}$	$\boldsymbol{\xi}$ – Posterior Mean (Standard Deviation)		
c^w	-1.651*** (0.024)	-0.578*** (0.029)	1.118*** (0.135)
$Answers\_received_{i,t-1}$	0.217*** (0.019)	0.021 (0.023)	0.013 (0.051)
$Upvotes\_answer_{i,t-1}$	0.231*** (0.016)	0.109*** (0.013)	0.031 (0.028)
$Accepted\_answers_{i,t-1}$	0.594*** (0.036)	0.235*** (0.023)	0.081*** (0.030)
$Badges_{i,t-1}$	0.400*** (0.037)	0.201*** (0.036)	-0.030 (0.058)
Initial Probability	0.755*** (0.015)	0.216*** (0.014)	0.029*** (0.005)

* $p < 0.1$, ** $p < 0.05$; *** $p < 0.01$. For brevity, we use "significant" and "insignificant" in the results discussion.

State-dependent Contributions (β)

We first examine the state-dependent contributions (top panel in Table 3). The interpretation of the three states is determined by the state-specific intrinsic propensity to contribute (the constant vector c^x), as discussed in the Estimation and Identification section. The estimates are -3.053, 0.316, and 7.072 for states L, M, and H, respectively (all significant at 1% level). The relative large distances between states indicate that the states are well identified.

The coefficients of *Matched_tags* are 0.015, 0.022, and 0.029 for states *L*, *M*, and *H*, respectively (all significant at 1% level). The positive coefficients suggest that all users tend to supply more knowledge when the need arises and matches their expertise. Also, the increasing magnitude of the coefficients shows that as users move from *L* to *M* to *H*, they become more responsive to the demand for knowledge.

For other individual characteristics, we find a negative relationship between *Tenure* length and user contributions. The negative coefficients in state *M* (-0.006, significant at 1% level) and state *H* (-0.016, significant at 1% level) suggest that users involved for a longer time tend to contribute less. It is possible that, all else being equal, the longer a user has been associated with the community, the more inertia (or lower incentives) he has in terms of contribution. Such a *stalling effect* poses another challenge to online communities.

We also confirm that the answers a user contributed in the past have a positive relationship with how much he would contribute in the future. The highly significant coefficients of *Total_answers* for state *L* (0.001), *M* (0.002), and *H* (0.003) show that the effect of this variable increases as users moves from state *L* up to state *H*.

Regarding *Group_size*, the coefficients are positive and significant in *L* (0.003) and *M* states (0.002). A user may contribute more when the community is larger, which confirms the "social effect" discussed before. Yet, this effect decreases as a user transitions from state *L* to *M*. As the user moves further up to state *H*, the community size is no longer a positive factor (-0.01, significant at 1%). This suggests that users in states *L* and *M* may put higher valuation on a larger community, while users in state *H* may be more susceptible to the free-riding problem.

State Transition Probabilities (ξ)

We now turn to the effects of different motivating mechanisms on state transition probabilities (bottom panel in Table 3). The constant term c^w is negative: -1.651 and -0.578 for states *L* and *M*, respectively (both significant at 1% level). This indicates that in the absence of motivating mechanisms, users in states *L* and *M* are likely to stay or transition to lower states. This once again reinforces the importance to have motivating mechanisms in place, or else the community will decline.

Also, the more negative coefficient for users in state *L* suggests that they are more likely to be in *L* compared with users in *M*. In contrast, the constant term c^w becomes positive in state *H* (1.118, significant at 1% level). Users in *H* state tend to remain highly motivated.

Overall, users in all three states seem to benefit from the motivating mechanisms through their interaction with the community. However, different mechanisms have different impacts on the motivation conditional on a user's current state. When comparing the coefficients corresponding to each motivating mechanism across the three states, we can see that the mechanisms are the most effective among the low-state users. This is also precisely the user state that needs to be activated by the online community. Such finer-grained results were not revealed in the prior literature.

Specifically, reciprocity seems to be effective only for the least motived users. For users in state *L*, receiving more answers on their previous questions tends to help transfer them into higher states (0.217, significant at 1% level). The coefficient becomes insignificant in states *M* and *H*. They tend to contribute anyway, less because they want to return the favour of their peers. Hence, *reciprocity* can be more useful to stimulate low-state users.

For peer-recognition, the coefficients on *Upvotes_answer* and *Accepted_answers* are all positive and mostly significant (rows 3-4 in the bottom panel). We interpret this result as the verification of one's identity. When a user receives more up-votes or has more answers accepted, he may feel the value of his contribution being recognized and thus his identity in the community validated. Further, this identity-verification effect is more prominent in lower states, and declines as a user moves to *H*. For instance, *Accepted_answers* motivates users in state *L* (0.594, significant at 1%) the most, followed by state *M* (0.235, significant at 1%), and then state *H* (0.081, significant at 1%). This indicates that the marginal effect of accepted answer diminishes as a user moves to higher motivation states. The pattern is similar for *Upvotes_answer*. Together, these results highlight the effectiveness of *peer recognition* as a motivating scheme to enhance self-identity, especially for users in low state.

Likewise, we find the effectiveness of *badges* to strengthen self-image motivation. Earning more badges seems to lift a user from states *L* and *M* to state *H* (coefficients are 0.400 and 0.201, significant at 1% level). The effect of *badges* becomes insignificant for users in *H*. This seems to suggest that highly motivated users are insensitive to badges; earning badges may not help them verify their self-identity. This may be due to the "moral licensing" effect of pro-social behavior. If so, using badge system to motivate user contributions should be gauged carefully, despite the fact that badges are widely used in many online communities. To retain users in *H* state, community managers need to design more effective mechanisms. This could be an interesting area for future research.

Transition Matrices and Marginal Effects

We substitute the estimates from Table 3 into equation (5) to calculate the transition probabilities among states. Transition matrix (a) in Table 4 presents the transition probabilities evaluated at the mean level of community interactions (from column "Mean" in Table 1). The transition probabilities are substantially different when a user is in *L*, *M* or *H* states. This confirms that modelling the stochastic process with the three hidden states is reasonable. Further, the matrix indicates the stickiness of state *L.* Once a user is in this state, he is most likely to be trapped, and even if a user starts off in state *H*, he also tends to slip down to *M* and then to *L.* This implies the challenge of inherent deteriorating participation as we posed earlier, and the importance of stimulating users to become more motivated.

Table 4. Mean Posterior Transition Matrices

	(a) Mean Interactions			(b) Up-votes		
t-1 to t	*L*	*M*	*H*	*L*	*M*	*H*
L	94.2	5.8	0.0	91.0	8.9	0.0
M	70.8	28.8	0.4	66.9	32.5	0.6
H	13.1	69.7	17.2	13.1	69.7	17.2
	(c) Accepts			(d) Badges		
t-1 to t	*L*	*M*	*H*	*L*	*M*	*H*
L	83.6	16.3	0.1	88.0	12.0	0.0
M	62.2	36.9	0.8	63.5	35.7	0.8
H	11.5	69.2	19.3	13.1	69.7	17.2

Note: All numbers are probabilities (%).

To quantify the marginal effect of each motivating mechanism on transition probability, we calculate the transition probabilities when the mean value of a variable increases by one unit, while holding other variables constant. The matrices (b) – (d) in Table 4 show the transition probabilities caused by such a change in *up-votes*, *accepts* and *badges*, respectively. We focus on *up-votes*, *accepts* and *badges*, because they are the mechanisms that platform designers could manage. For example, if the community decreases the cost of up-votes or even enhances the incentives of up-votes, the number of up-votes is likely to increase. If the platform designer changes the setup such that each question could accept multiple answers, then the mean of accepted answers is likely to increase. Further, because online communities provide various kinds of badges to users, a more careful design of the badge system may help elevate the user contributions.

We can then take the difference between respective cells of (a) and (b) – (d) to calculate the marginal effect on transition probability. For example, in matrix (b), receiving one additional *up-vote* on average hypothetically increases the probability of transitioning from state *L* to state *M* by 3.1% (from 5.8% to 8.9%), while a user in state *M* would increase his likelihood of staying in the state from 28.8% to 32.5%, and that of switching to state *H* from 0.4% to 0.6%. Similarly, in matrix (c), one additional *accepted answer* could increase the transition probability from state *L* to state *M* by 10.5% (from 5.8% to 16.3%), and increase the probability of staying in state M by 8.1% (from 28.8% to 36.9%). It also increases the transition probability to state *H* by 0.1%, 0.4% and 2.1% for users in state *L*, *M*, and *H*, respectively. Such changes are non-trivial because the low motivation state tends to be sticky. With more than 200 up-votes and 30 accepted answers each day on SuperUser, the effects of these mechanisms significantly enhance the contributions at the community level.

Design Simulations

We now turn to the normative perspective on motivating mechanisms using design simulations.[8] We do three simulation experiments to see if platform designers can encourage more contributions by strengthening users' internalized extrinsic motivations through calibrating specific IT-artifacts: *up-votes* received for answers, *accepted answers*, and *badges*. If it becomes easier to enhance peer recognition and self-image through each of these channels, are users going to provide more answers? We hypothetically double the value of the variables *Upvotes_answer*, *Accepted_answers*, and *Badges*, making it twice as easy to earn each reward. We then simulate, under each scenario, the evolution of the total number of answers and users in state M and state H over time.

Figure 5 presents the results, which are the average of 100 simulation iterations for each user on each date. The first column (graphs 1, 4, and 7) shows the simulated total number of answers (grey dots) versus the actual total number of answers (black dots) for the doubled *Upvotes_answer*, *Accepted_answers*, and *Badges*, respectively. To better illustrate the effects and trends, we fit the simulated answers with a dash curve and the actual answers with a solid curve. Both curves are smoothed. In the second column (graphs 2, 5, and 8), we plot the

[8] These simulation experiments correspond to "counterfactual experiments" in the empirical industrial organization literature (Reiss and Wolak 2007). In a structural model, if we specify a counterfactual antecedent (an event/parameter different from the real observations), then we can evaluate the counterfactual consequent (a result that is expected to hold if the antecedent were true). Such analysis is often used for policy evaluation.

number of users who are in state *M*. Similarly, the solid line shows users in state *M* under the current design, and the dashed lines show simulated users in state *M* if we were to change the corresponding motivation mechanism. In the third column, we plot the number of users who are in state *H* in a similar way.

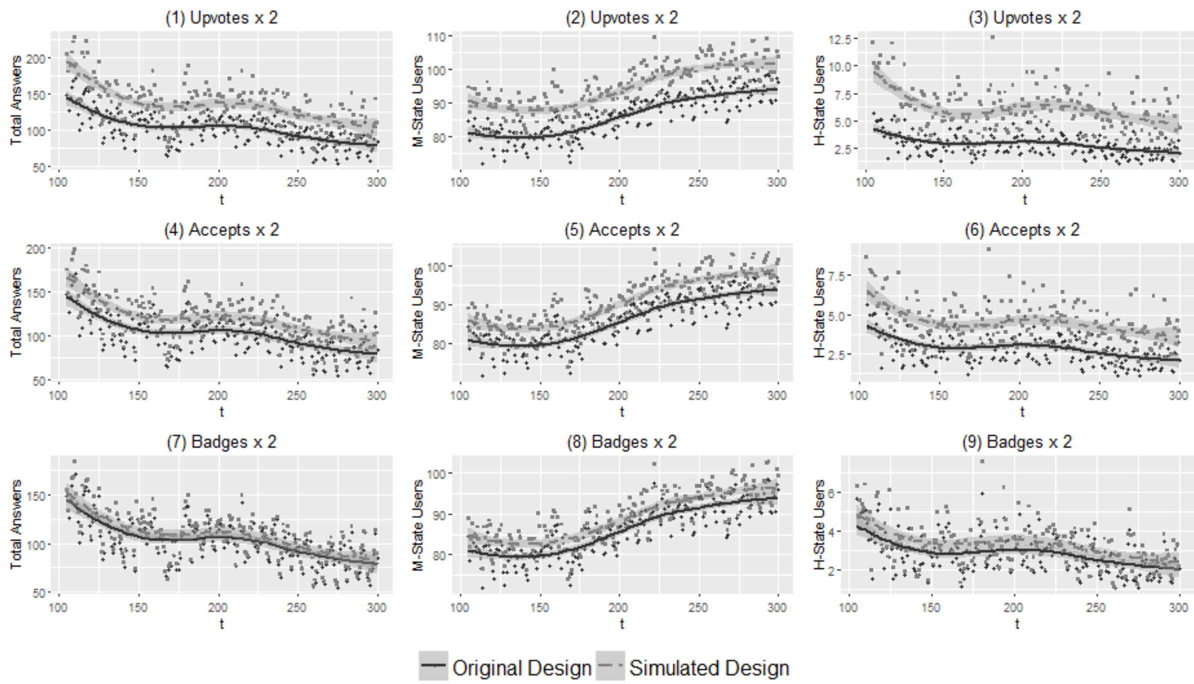

Figure 5. Design Simulations: What if rewards are easier to earn?

We discover three patterns here. First, the simulated number of answers is greater than the actual data in all three cases. This means that when it becomes easier to receive rewards to enhance one's internalized extrinsic motivation (through up-votes, accepted answers or badges), users will contribute more. Second, up-votes and accepted answers seem to be more effective than badges, which may be due to the "moral licensing" effect of badges in high motivation state. However, as a design mechanism, badges are much easier to change than up-votes and accepted answers. To test the effectiveness of different badges, a platform designer could potentially examine the simulated experiment on many specific badges. Third, highly motivated users are critical to the knowledge contribution and accumulation in the online community. The actual number of users in state *H* is relatively small, and it also declines slightly. The stimulation of more up-votes, accepted answers, and badges not only boost the number of these users in state *H*, but also smooth the declining trend.

Together, our experiments suggest that it is important for platform designers to manage internalized extrinsic motivation, so as to encourage users to contribute more. Note that we are not suggesting a constant effect of these mechanisms; as we change the design of the community, the perception of the users may change accordingly. Rather, our design simulation opens up a direction for further exploration.

Robustness Checks

We conduct several robustness checks and provide the detailed results in Appendix A4. First, to ensure that our results are not biased by the sample period, we estimate the model on several alternative steady-state sample periods (e.g., 301-500 days). The results from those estimations are consistent to the results reported above. We also examine the first 100 days, where there is substantial fluctuation of user activities. Compared with the steady periods, the magnitude of coefficients across states do not have an evidently different pattern.

Second, to probe deeper into the possible heterogeneity of users and questions, we test the following variables in our model: 1) Special roles of users. Some users are elected as moderators, which may explain their higher contribution. We include a binary indicator of whether a user is a moderator, and find its coefficient is only significant in states *M* and *H*. Hence, on average a moderator is more likely to be in higher motivation states. 2) Questions from influential users. Questions from influence users, such as moderators, can motivate users to contribute. We construct a variable to measure how many questions are asked by moderators. The coefficient on this variable is insignificant across all states, suggesting that the overall contribution of users is insensitive to whether or not the question comes from influential individuals. These analyses provide additional insights, while our main results remain consistent as before.

Third, we conduct robustness checks on different time aggregations and user samples. We aggregate daily data into weekly to smooth any possible fluctuation during a week (e.g., weekday vs. weekend effect). Results on weekly data are consistent with the daily analysis. To alleviate computational burden, results presented so far are based on a sample with users that have contributed more than 10 answers during the sample period. We also estimate the model separately on all users on the site, users who have contributed at least one answer, and those with more than 50 answers, respectively. The results are consistent.

Conclusions

User contributions are voluntary but vital in many online communities. This paper studies the effects of motivating mechanisms on voluntary contributions through IT-artifacts design from a *dynamic* perspective. Using a hidden Markov model under the public goods framework, we identify three motivation states that increase in the propensity of contribution, and investigate the effect of several types of mechanisms (reciprocity, peer recognition, and self-image) on transitioning users between the states.

This dynamic perspective is a unique feature of our work. The existing literature relies on a conventional static approach, which implicitly assumes that the relationship between motivating mechanisms and user contributions is *static*. In contrast, our dynamic approach allows the effect of motivating mechanisms to change across users and over time. As such, our approach advances the literature on voluntary user contributions in online communities. It also enriches the literature on public goods that features in impure altruism and internalized extrinsic motivations.

Our results from the dynamic model shed light on a key question in the research on online communities, i.e., how to *design* effective IT artifacts that can engage users to contribute. We find that IT artifacts (e.g., up-votes, accepted answers, and badges) as motivational devices are useful to elevate contributions, but their influence varies significantly with motivation states. Although badges are widely used in practice, they can be ineffective for users in high motivation state. Hence, the design of badge system (or gamification in a broader sense) deserves careful consideration. In contrast, up-votes and accepted answers are shown to be much more effective across motivation states. These results highlight the effectiveness of *peer recognition* as a motivating scheme, especially in switching users to and retaining them in higher motivation states.

Our results provide important managerial implications for devising various mechanisms, and evaluating their effectiveness on encouraging user contribution. First, managers need to be mindful that users have a different propensity to contribute, and it is crucial to design proper instruments to motivate contributions. The changing influence of peer-recognition in different states suggests that community managers can gear their intervention towards users in specific motivation states. For example, if the goal is to induce more users to be in high motivation state, then encouraging users to accept high quality answers may be a more effective intervention than adding more badges. Second, our structural model allows community managers to perform interesting design simulations and evaluate the consequence of changing certain mechanisms. As our design simulations suggest, if the design of the community makes it easier for users to gain up-votes or accepted answers, for example by allowing users to accept more than one answer of high quality, then users are more likely to become highly motivated. A platform designer can also experiment with a specific badge and decide how to adjust it. Third, managers should also consider how to foster the community. It would be helpful to attract new users to make the "social effect" more prominent, and encourage users to ask questions to raise the "demand of knowledge." The estimated hidden states from our model allow community managers to classify users in real time and help them target the right kinds of users.

Our *dynamic* framework would help motivate future work on the design of IT artifacts in online communities. First, while we use a knowledge sharing community as a testing field, our framework is applicable to many other online communities relying on *voluntary* user contributions. One may caution that there are other design features unavailable in our contexts. However, our *dynamic* framework, with appropriate modifications under specific research context (especially the utility function), can be extended to other settings. Second, our findings point to several interesting avenues of future research. For instance, the insignificant effect of badges in high motivation state implies that there may be too many trivial badges in the system. It would be interesting to identify the types of badges that are truly effective. Another example is that from our analysis, we cannot conclude the changing effectiveness of motivating mechanisms across the lifecycle of the community. A more in-depth study along this line may generate interesting implications in the future.

More broadly, this research is related to open models of co-production in user communities over time. Our study informs the design of internalized extrinsic mechanisms in contexts such as crowdsourcing, user generated contents, open innovation, and even crowdfunding. As data become available, future research may expand into these broader areas. In such environments, production moves beyond the boundary of traditional, formal organizational structure, and voluntary contributions facilitated by IT artifacts become crucial. New challenges such as the design of motivating mechanisms in a dynamic environment need to be managed, so that the co-production of open communities can be sustainable (Zhu and Zhou 2012). Our paper, although currently examined in a knowledge sharing community, may generate new insights into designing IT artifacts to engage voluntary contributions in online communities without formal governance or compensation structures in the traditional sense. While many open questions remain, we hope our analysis framework and initial findings will help stimulate more research in this growing area.

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ONLINE APPENDIX

ENGAGING VOLUNTARY CONTRIBUTIONS IN ONLINE COMMUNITIES: A HIDDEN MARKOV MODEL

A1. The MCMC Estimation of the HMM

We estimate the parameters vector $\{\theta, \tilde{S}\}$ with Gibbs sampling (Albert and Chib 1993). Suppose we have motivation state $s_{it} \in \{1, 2, \dots, J\}$ in our model. We generate the joint posterior distribution by sampling from each conditional distribution of the following parameter blocks:

$$\boldsymbol{\theta} = (\theta_1, \theta_2', \theta_3', \theta_4', \theta_5')'$$
$$\theta_1 = \sigma^{-2}$$
$$\theta_2 = \left(\beta_1', \beta_2', \dots, \beta_J'\right)'$$
$$\theta_3 = \left(\xi_1', \xi_2', \dots, \xi_J'\right)'$$
$$\theta_4 = \left(\mu_2, \mu_3, \dots, \mu_{J-1}\right)'$$
$$\theta_5 = (L_{it}, L_{i2}, \dots, L_{iT})', i = 1, \dots, n$$
$$\theta_6 = (s_{i1}, s_{i2}, \dots, s_{iT})', i = 1, \dots, n$$

For the simplicity of presentation, we denote $\theta_{-i} = \left(\theta_j'\right)', \forall j \neq i$ below.

(1) Sample $\theta_1 = \sigma^{-2}$ **from** $P(\theta_1 | \theta_{-1}, Y, X, W)$.

Prior: $\sigma^{-2} \sim \Gamma\,(\alpha, \delta)$. Conditional on θ_{-1}, Y, X, and W, it is equivalent to observing $\{\varepsilon_{\mathrm{it}}\}$ where $\varepsilon_{it} = Y_{it} - X_{it}\beta_{s_{it}}$.

Posterior: $(\sigma^{-2} | \theta_{-1}, Y, X, W) \sim \Gamma\,(\alpha + \frac{1}{2}nT, \delta + \frac{1}{2}SSR)$, where $SSR = \sum_{i=1}^{n}\sum_{t=1}^{T}\varepsilon_{it}^2$.

(2) Sample $\theta_2 = \left(\beta_1', \beta_2', \dots, \beta_J'\right)'$ **from** $P(\theta_2 | \theta_{-2}, Y, X, W)$.

Prior: $\left(\beta_{\mathrm{j}} | \sigma^{-2}\right) \sim N(m_j, M_j),\ j = 1, \dots, J$ (independent of each other)

Posterior: Conditional on $\{s_{it}\}$, only those observations for which $s_{it} = j$ are relevant to posterior distribution of β_j: $\left(\beta_{\mathrm{j}} | \theta_{-2}, Y, X, W\right) \sim N(m_j^*, M_j^*)$, where

$$M_j^* = \left(M_j^{-1} + \sigma^{-2}\sum_{i=1}^{n}\sum_{t=1}^{T} X_{it}X_{it}'\, 1_{\{s_{it}=j\}}\right)^{-1}$$

and

$$m_j^* = M_j^*\left(M_j^{-1}m_j + \sigma^{-2}\sum_{i=1}^{n}\sum_{t=1}^{T} X_{it}Y_{it}\, 1_{\{s_{it}=j\}}\right).$$

(3) Sample $\theta_3 = \left(\xi_1', \xi_2', \dots, \xi_J'\right)'$ **from** $P(\theta_3 | \theta_{-3}, Y, X, W)$.

Prior: $\xi_j \sim N(mw_j, Mw_j),\ j = 1, \dots, J$.

Posterior: $\left(\beta_{\mathrm{j}} | \theta_{-3}, Y, X, W\right) \sim N(mw_j^*, Mw_j^*)$, where

$$Mw_j^* = \left(Mw_j^{-1} + \sum_{i=1}^{n} \sum_{t=1}^{T} W_{i,t-1} W'_{i,t-1} \, 1_{\{s_{i,t-1}=j\}} \right)^{-1}$$

and

$$mw_j^* = Mw_j^* \left(Mw_j^{-1} mw_j + \sum_{i=1}^{n} \sum_{t=1}^{T} W_{i,t-1} L_{it} \, 1_{\{s_{i,t-1}=j\}} \right).$$

Note that since σ_u^2 is not identifiable, we normalize it to 1 in the estimation.

(4) Sample $\theta_4 = (\mu_2, \mu_3, \ldots, \mu_{J-1})'$ **from** $P(\theta_4 | \theta_{-4}, Y, X, W)$.

Albert and Chib (1993) provide the posterior for μ_j given the other threshold parameters μ_k, $k \neq j$. For each μ_j, let $Lower = \max\{\max\{L_{it}: s_{it} = j\}, \mu_{j-1}\}$ and $Upper = \min\{\min\{L_{it}: s_{it} = j+1\}, \mu_{j+1}\}$. Then we can sample μ_j from the uniform distribution $U[Lower, Upper]$.

(5) Sample $\theta_5 = (L_{i1}, L_{i2}, \ldots, L_{iT})', i = 1, \ldots, n$ **from** $P(\theta_5 | \theta_{-5}, Y, X, W)$.

L_{it} determines s_{it} according to the following formula:

$$s_{it} = j \; if \; \mu_{j-1} < L_{it} < \mu_j,$$

where $\mu_0 = -\infty$, $\mu_1 = 0$, $\mu_J = \infty$, and $\mu_2, \ldots, \mu_{J-1}$ are given in step (4). Conditional on θ_{-5}, we can generate L_{it} from a truncated normal distribution

$$TN_{(\mu_{j-1}, \mu_j)}(W_{i,t-1} \xi_{s_{i,t-1}}, 1),$$

which is a normal distribution with mean $W_{i,t-1} \xi_{s_{i,t-1}}$ and variance 1, and truncated left at μ_{j-1} and right at μ_j. Repeating this for $t = 1, \ldots, T$ and $i = 1, \ldots, n$ gives a draw from $P(\theta_5 | \theta_{-5}, Y, X, W)$.

(6) Sample $\theta_6 = (s_{i1}, s_{i2}, \ldots, s_{iT})', i = 1, \ldots, n$ **from** $P(\theta_6 | \theta_{-6}, Y, X, W)$.

We generate the states using the single-move Gibbs-sampling algorithm in Kim and Nelson (1999), which is also the well-known Forward-Backward algorithm. Denoting Ψ_{it} as information for user i up to time t, and Ψ_{iT} as information from the whole sample, we follow the forward-backward algorithm as below to obtain $P(s_{it} | S_{i,-t}, \Psi_{iT})$:

(a) **Forward**: Calculate $P(s_{it} | \Psi_{it})$.

Step 1: Given $P(s_{i,t-1} = k | \Psi_{i,t-1})$, $k = 1, \ldots, J$ at the beginning of period t, calculate $P(s_{it} = j, s_{i,t-1} = k | \Psi_{i,t-1}) = P(s_{it} = j | s_{i,t-1} = k, \Psi_{i,t-1}) P(s_{i,t-1} = j | \Psi_{i,t-1})$, where

$$P(s_{it} = j | s_{i,t-1} = k, \Psi_{i,t-1}) = \begin{cases} \Phi(\mu_1 - W_{i,t-1}\xi_k), & if \; j = 1 \\ \Phi(\mu_j - W_{i,t-1}\xi_k) - \Phi(\mu_{j-1} - W_{i,t-1}\xi_k), & if \; j = 2, \ldots, J-1 \\ 1 - \Phi(\mu_{j-1} - W_{i,t-1}\xi_j), & if \; j = J \end{cases}.$$

For the first period, we use the initial probability $P(s_{i1} = j) = p_j$ for $j = 1, \ldots, J$, which are sampled from a Dirichlet distribution.

Step 2: Once X_{it} and Y_{it} are observed in period t, we update the probability term by calculating $P(s_{it} = j|\Psi_{it}) = \sum_{k=1}^{J} P(s_{it} = j, s_{i,t-1} = k|\Psi_{it})$, where

$$
\begin{aligned}
&P\left(s_{it} = j, s_{i,t-1} = k \mid \Psi_{it}\right) \\
&= P\left(s_{it} = j, s_{i,t-1} = k \mid \Psi_{i,t-1}, X_{it}, Y_{it}\right) \\
&= \frac{f\left(Y_{it}\middle|s_{it} = j, s_{i,t-1} = k, \Psi_{i,t-1}, X_{it}\right)P\left(s_{it} = j, s_{i,t-1} = k\middle|\Psi_{i,t-1}\right)}{f\left(Y_{it}\middle|\Psi_{i,t-1}, X_{it}\right)} \\
&\propto f(Y_{it}|s_{it} = j, X_{it})P\left(s_{it} = j, s_{i,t-1} = k\middle|\Psi_{i,t-1}\right).
\end{aligned}
$$

(b) **Backward**: In the backward process, we generate s_{it} conditioning on Ψ_{it} and $s_{i,t+1}$ $(t = T-1, T-2, \ldots, 1)$ using $g(s_{it}|\Psi_{it}, s_{i,t+1}) \propto g\left(s_{i,t+1}\middle|s_{it}, \Psi_{it}\right)g(s_{it}|\Psi_{it})$. We then can calculate

$$
P\left(s_{it} = j\middle|s_{i,t+1}, \Psi_{it}\right) = \frac{g\left(s_{i,t+1}\middle|s_{it} = j, \Psi_{it}\right)g(s_{it} = j|\Psi_{it})}{\sum_{k=1}^{J} g\left(s_{i,t+1}\middle|s_{it} = k, \Psi_{it}\right)g(s_{it} = k|\Psi_{it})}.
$$

Then we can use a random number drawn from a uniform distribution to generate s_{it} according to $P(s_{it}|S_{i,-t}, \Psi_{iT})$.

A2. Log-likelihood and Model Selection Criteria

As detailed in Appendix A1, we estimate the parameters in our HMM with Bayesian estimation, which does not require us to calculate the likelihood. However, to select the number of states, the selection criteria would rely on the likelihood. Therefore, we describe the calculation of the likelihood of an observed sequence of contributions and the selection criteria below.

Log-likelihood Calculation

Because we adopt a hidden Markov model, the contribution probabilities for each individual over time are correlated through the hidden states. The joint likelihood of each individual's contribution sequence has to consider the possible paths of the underlying states (Netzer et al. 2008). Suppose that there are J possible states. Then according to MacDonald and Zucchini (1997), we can write the joint probability using a matrix product as

$$P_i(Y_{i1} = y_{i1}, \ldots, Y_{iT} = y_{iT}) = P_0 \Omega_i(1) Q_i(1,2) \Omega_i(2) \cdots Q_i(T-1,T) \Omega_i(T) \mathbf{1}',$$

where P_0 is the initial probability, $\Omega_i(t)$ is a $J \times J$ diagonal matrix with the elements of emission probability $\omega_{it|j} = f(Y_{it}|X_{it}, s_{it} = j; \boldsymbol{\beta}, \sigma^2)$ on the diagonal, $Q_i(t-1,t)$ is the $J \times J$ transition matrix for individual i at time t with the elements of $q_i(k,j) = f\left(s_{it} = j \middle| W_{i,t-1}, s_{i,t-1} = k; \boldsymbol{\xi}\right)$ on the k^{th} row and j^{th} column, and $\mathbf{1}'$ is a $J \times 1$ vector of ones. The element probabilities are obtained according to our model setup:

$$\omega_{it|j} = f(Y_{it}|X_{it}, s_{it} = j; \boldsymbol{\beta}, \sigma^2) = \left\{1 - \Phi\left(\frac{X'_{it}\beta_j}{\sigma}\right)\right\}^{1\{Y_{it}=0\}} \left\{\frac{1}{\sigma}\phi\left(\frac{Y_{it} - X'_{it}\beta_j}{\sigma}\right)\right\}^{1\{Y_{it}>0\}},$$

and

$$q_i(k,j) = f\left(s_{it} = j \middle| W_{i,t-1}, s_{i,t-1} = k; \boldsymbol{\xi}\right) = \Phi\left(\mu_{j+1} - W_{i,t-1}\xi_k\right) - \Phi\left(\mu_j - W_{i,t-1}\xi_k\right).$$

Then we can write the log-likelihood as $\ln L = \sum_i \log(P_i)$.

Selection Criteria

We adopt three model selection criteria to determine the number of states in our HMM. First, we use the commonly used Akaike information criterion (AIC) and Bayesian information criterion (BIC) (Singh et al. 2011, Yan and Tan 2014):

$$AIC = -2 * \ln L + 2 * size,$$

and

$$BIC = -2 * \ln L + size * \ln N,$$

where *size* is the number of parameters in the model, and N is the number of users in the sample. Second, realizing that we are using a Bayesian estimation for our HMM, we also adopt Markov switching criterion (MSC), which was developed for HMM's state and variable selection (Smith et al. 2006). We follow the adaptation in the literature for its formulation (Netzer et al. 2008):

$$MSC = -2 * \ln L + \sum_{s=1}^{J} \frac{\hat{T}_s\left(\hat{T}_s + J * K\right)}{\hat{T}_s - J * K + 2},$$

where $\hat{T}_s = \sum_{t=1}^{T} \sum_{i=1}^{N_t} P(s_{it} = s)$, J is the number of states in the model, and K is the number of covariates in both the transition matrix and the state-dependent vector.

A3. Testing the Estimation on Simulated Data

Because our model has a non-linear feature by incorporating the *Tobit* and *probit* models, we could not use standard statistical software to estimate it. We have to write our own estimation algorithm instead. Hence we did, but we need to ensure that it is correct before applying the algorithm to the actual data. We run the algorithm on simulated data based on known parameters, and test whether it could recover the "true" parameters. Because there is some model uncertainty on the number of states in our HMM, we also simulate data with 2, 3, and 4 "true" states, and then estimate the model with 2, 3, and 4 states in HMM. Then we use the model selection criteria to determine whether our algorithm points out the "true" number of states. Here we use three "true" states as an example.

We first generate the "true" parameters $\boldsymbol{\theta}$, the community and individual characteristics variables $X = \{X_{it}\}_{t=1,\ldots,T;i=1,\ldots,N_t}$, and the community interaction variables $W = \{W_{it}\}_{t=1,\ldots,T;i=1,\ldots,N_t}$ with three motivation states ($J = 3$). Since we assume that a user has an initial probability $P_0 = \{p_1, p_2, p_3\}$, at $t = 1$ we draw the initial state s_{i1} of user i from a Dirichlet distribution using the initial probability P_0 for each user i that enters the community. Conditional on s_{i1}, we then draw the contribution $Y_{i1} = \max(0, Y_{i1}^*)$, where $Y_{i1}^* = X_{it}\beta_{s_{i1}} + \varepsilon_{i1}$ and ε_{i1} is generated from a normal distribution with mean 0 and variance σ^2. For any $t > 1$, we first draw $L_{it} = W_{i,t-1}\xi_{s_{i,t-1}} + u_{it}$, where u_{it} is drawn from $N(0, 1)$. Then we generate the new state s_{it} according to L_{it}. Repeating the same process, we generate $Y = \{Y_{it}\}_{t=2,\ldots,T;i=1,\ldots,N_t}$ for all t.

With the simulation data $\{X, W, Y\}$, we estimate the model with our procedure and present the results in Table A3.1. Our simulation data contains 322 individuals and 20 periods of time. The true number of states is $J = 3$. The community and individual characteristics vector X contains four variables, and the community interaction vector W contains four variables. In Table A3.1, the "True Parameters" panel on the left displays the original parameters $\theta = \{\beta, \xi, \sigma^2\}$ that we employ to generate the simulation data. The "Estimation" column on the right displays the estimated parameters. Our estimation recovers the "true" parameters accurately.

We also present the model selection criteria in Table A3.2. Given the "true" state number is three, all our model selection criteria indicate that our HMM model with three states fit the data the best. This confirms the reliability of the estimation algorithm, and gives us confidence in its empirical application to the actual data.

Table A3.1. Estimation Results from Simulation Data (Number of States = 3)

	True Parameters			Estimation		
Variables	State 1	State 2	State 3	State 1	State 2	State 3
$\boldsymbol{\beta}$					Mean (Standard Deviation)	
x_1	3	5	7	2.98 (0.07)	4.99 (0.07)	6.99 (0.03)
x_2	4	6	8	4.00 (0.02)	6.02 (0.01)	8.01 (0.01)
x_3	5	7	9	4.99 (0.02)	7.00 (0.02)	8.98 (0.01)
x_4	6	8	10	6.00 (0.02)	8.00 (0.01)	10.01 (0.01)
σ^2		1.5			1.53 (0.03)	
$\boldsymbol{\xi}$						
w_1	-1.5	-0.5	0.5	-1.63 (0.13)	-0.48 (0.08)	0.42 (0.06)
w_2	1.15	0.37	2.53	1.18 (0.11)	0.27 (0.08)	2.57 (0.14)
w_3	6.32	4.48	7.35	6.53 (0.29)	4.41 (0.22)	7.67 (0.44)
w_4	2.65	3.05	6.96	2.73 (0.15)	3.10 (0.12)	7.04 (0.17)
μ_j		2			1.95 (0.04)	
σ_u^2		1			1	
$P_0 = \{p_j\}$	0.45	0.40	0.15	0.44 (0.02)	0.41 (0.03)	0.15 (0.020)
T = 20	*N* = 322	Draws = 2,000				

Table A3.2. Selection of Number of States from Simulation Data (Number of States = 3)

Number of States	- 2*Log-likelihood	AIC	BIC	MSC	Number of Variables
2	35404.87	35442.87	35514.59	41380.24	19
3	**22700.68**	**22758.68**	**22868.14**	**22700.68**	**29**
4	22734.37	22812.37	22959.57	30211.52	39

A4. Robustness Checks

We conduct several sets of robustness checks. First, we estimate the model on another sample period (301-500 days). The results are in Table A4.1. Second, we examine whether the moderator role of a user or new questions by the moderators in the community would affect the transition probability of a user. We control for these two factors separately in W_{it}, and present the results in Table A4.2 and Table A4.3, respectively. Lastly, we estimate the model on weekly data and include the results in Table A4.4.

Table A4.1. Results of HMM on Daily Data for Day 301-500 Subsample

Variable Name	State 1 (Low Motivation)	State 2 (Medium Motivation)	State 3 (High Motivation)
$\boldsymbol{X}_{it}$		β – Posterior Mean (Standard Deviation)	
c^x	-2.791*** (0.071)	-0.103 (0.091)	4.735*** (0.330)
$Matched\_tags_{it}$	0.015*** (0.001)	0.029*** (0.001)	0.046*** (0.002)
$Group\_size_t$	0.001*** (0.000)	0.006*** (0.000)	0.013*** (0.002)
$Tenure_{it}$	-0.001*** (0.000)	-0.004*** (0.000)	-0.01*** (0.001)
$Total\_answers_{i,t-1}$	0.0002*** (0.000)	-0.0003*** (0.000)	-0.001*** (0.000)
σ^2		1.009*** (0.005)	
$\boldsymbol{W}_{i,t-1}$		ξ – Posterior Mean (Standard Deviation)	
c^w	-1.752*** (0.035)	-0.644*** (0.043)	0.941*** (0.114)
$Answers\_received_{i,t-1}$	0.268*** (0.019)	0.049* (0.026)	-0.117 (0.077)
$Upvotes\_answer_{i,t-1}$	0.262*** (0.020)	0.117*** (0.014)	0.021 (0.028)
$Accepted\_answers_{i,t-1}$	0.562*** (0.037)	0.300*** (0.024)	0.084** (0.035)
$Badges_{i,t-1}$	0.250*** (0.053)	0.275*** (0.040)	-0.095* (0.058)
Initial Probability	0.797*** (0.017)	0.186*** (0.016)	0.017*** (0.004)

* $p < 0.1$, ** $p < 0.05$; *** $p < 0.01$.

Table A4.2. **Results of HMM after Controlling Moderator in *W***

Variable Name	State 1 (Low Motivation)	State 2 (Medium Motivation)	State 3 (High Motivation)
$\boldsymbol{X}_{it}$		β – Posterior Mean (Standard Deviation)	
c^x	-3.075*** (0.069)	0.325*** (0.087)	7.054*** (0.330)
$Matched\_tags_{it}$	0.015*** (0.001)	0.023*** (0.001)	0.030*** (0.001)
$Group\_size_t$	0.003*** (0.000)	0.002*** (0.000)	-0.010*** (0.002)
$Tenure_{it}$	-0.0004* (0.000)	-0.006*** (0.000)	-0.016*** (0.001)
$Total\_answers_{i,t-1}$	0.0005*** (0.000)	0.001*** (0.000)	0.003*** (0.000)
σ^2		1.010*** (0.006)	
$\boldsymbol{W}_{i,t-1}$		ξ – Posterior Mean (Standard Deviation)	
c^w	-1.655*** (0.018)	-0.581*** (0.026)	1.069*** (0.127)
$Answers\_received_{i,t-1}$	0.214*** (0.017)	0.021 (0.023)	0.013 (0.054)
$Upvotes\_answer_{i,t-1}$	0.238*** (0.023)	0.101*** (0.022)	0.015 (0.059)
$Accepted\_answers_{i,t-1}$	0.588*** (0.038)	0.221*** (0.036)	0.060 (0.096)
$Badges_{i,t-1}$	0.400*** (0.033)	0.198*** (0.033)	0.026 (0.063)
$Moderator_{i,t-1}$	-0.115 (0.086)	0.284*** (0.089)	0.803*** (0.145)
Initial Probability	0.758*** (0.014)	0.213*** (0.014)	0.029*** (0.005)
* $p < 0.1$, ** $p < 0.05$; *** $p < 0.01$.			

Table A4.3. **Results of HMM after Controlling New Questions by Moderator in *W***

Variable Name	State 1 (Low Motivation)	State 2 (Medium Motivation)	State 3 (High Motivation)
$\boldsymbol{X}_{it}$		β – Posterior Mean (Standard Deviation)	
c^x	-2.990*** (0.080)	0.364*** (0.079)	7.368*** (0.365)
$Matched\_tags_{it}$	0.015*** (0.000)	0.023*** (0.001)	0.029*** (0.001)
$Group\_size_t$	0.002*** (0.001)	0.002*** (0.000)	-0.011*** (0.002)
$Tenure_{it}$	-0.0005** (0.000)	-0.006*** (0.000)	-0.015*** (0.001)
$Total\_answers_{i,t-1}$	0.0005*** (0.000)	0.001*** (0.000)	0.003*** (0.000)
σ^2		1.012*** (0.005)	
$\boldsymbol{W}_{i,t-1}$		ξ – Posterior Mean (Standard Deviation)	
c^w	-1.666*** (0.020)	-0.567*** (0.027)	1.442*** (0.246)
$Answers\_received_{i,t-1}$	0.220*** (0.016)	0.014 (0.023)	0.003 (0.059)
$Upvotes\_answer_{i,t-1}$	0.234*** (0.015)	0.115*** (0.014)	0.028 (0.027)
$Accepted\_answers_{i,t-1}$	0.592*** (0.035)	0.253*** (0.026)	0.075** (0.036)
$Badges_{i,t-1}$	0.408*** (0.034)	0.221*** (0.038)	-0.049 (0.064)
$New\_q\_moderators_{i,t-1}$	-0.002 (0.005)	-0.003 (0.009)	0.001 (0.038)
Initial Probability	0.758*** (0.016)	0.214*** (0.015)	0.028*** (0.005)
* $p < 0.1$, ** $p < 0.05$; *** $p < 0.01$.			

Table A4.4. Results of HMM on Weekly Data for Day 101-300 Subsample

Variable Name	State 1 (Low Motivation)	State 2 (Medium Motivation)	State 3 (High Motivation)
$\boldsymbol{X}_{it}$		β – Posterior Mean (Standard Deviation)	
c^x	-0.986*** (0.116)	10.566*** (0.772)	37.557*** (2.661)
$Matched\_tags_{it}$	0.003*** (0.000)	0.014*** (0.001)	0.031*** (0.001)
$Group\_size_t$	0.002*** (0.001)	-0.062*** (0.005)	-0.213*** (0.016)
$Tenure_{it}$	-0.002*** (0.000)	-0.029*** (0.001)	-0.070*** (0.003)
$Total\_answers_{i,t-1}$	0.002*** (0.000)	0.013*** (0.000)	0.024*** (0.001)
σ^2		2.028*** (0.030)	
$\boldsymbol{W}_{i,t-1}$		ξ – Posterior Mean (Standard Deviation)	
c^w	-1.835*** (0.043)	-0.793*** (0.066)	-0.124*** (0.303)
$Answers\_received_{i,t-1}$	0.072*** (0.012)	0.023 (0.015)	0.021 (0.035)
$Upvotes\_answer_{i,t-1}$	0.069*** (0.017)	0.018 (0.017)	0.013 (0.027)
$Accepted\_answers_{i,t-1}$	0.110*** (0.038)	0.056*** (0.018)	0.128 (0.083)
$Badges_{i,t-1}$	0.183*** (0.033)	0.144*** (0.036)	-0.054 (0.044)
Initial Probability	0.769*** (0.045)	0.192*** (0.041)	0.039*** (0.011)

* $p < 0.1$, ** $p < 0.05$; *** $p < 0.01$.